Title: Just a Matter of Time

Brief Overview:

Students will perform an experiment to investigate how varying lengths of a pendulum transform a sinusoidal curve. They will also investigate the relationship between algebraic models, graphical representations, and physical phenomena.

Link to Standards:

- **Problem Solving** Students will demonstrate their ability to model a real-world phenomena from the variety of functions obtained in the pendulum experiment.
- **Communication** Students will interact and discuss transformations of sinusoidal curves.
- **Reasoning** Students will interpret data obtained from a swinging pendulum to logically determine what effect the length of the pendulum has on one cycle.
- **Connections** Students will recognize and demonstrate that trigonometric functions, graphic models, and physical models are interrelated.
- **Trigonometry** Students will write equations for sinusoidal curves from data that represents a swinging pendulum.

Grade/Level:

Grades 10–12; Trigonometry/Pre-Calculus/Calculus

Duration/Length:

This lesson is expected to take one to two 45 minute class periods.

Prerequisite Knowledge:

Students must be familiar with writing and interpreting equations of sinusoidal curves in standard form, and use of the TI-82 graphics calculator.

Objectives:

Students will be able to:

- work cooperatively in groups.
- collect and organize data from resources.
- analyze the amplitude, period, and vertical & phase shifts of sinusoidal curves, and then write equations.
- become familiar with linking the TI-82, the CBL unit, and the motion detector probe.
- relate algebraic, graphic, and physical models.
- interpret collected data, and apply to new situations.

Materials/Resources/Printed Materials:

Each group will need the following:

- CBL unit
- TI-82 graphics calculator with unit-to-unit link cable
- Vernier CBL Motion Detector
- Program TICTOC[1]
- STUDENT INSTRUCTION SHEET
- STUDENT ACTIVITY SHEET
- IN CONCLUSION... (worksheet)
- Pendulum unit consisting of string and empty soda can
- Stopwatch OR digital watch
- Tape
- Meter stick
- Three chairs & desks

NOTE: If a limited number of CBL units and/or probes are available, the teacher may want to use this lesson for demonstration only (as whole group), and the entire class would record and analyze the same data.

Development/Procedures:

- Give students a general idea of the experiment and what to expect.

- Make sure each student has a copy of the TICTOC program. (This can be done by using the cable link to send the program from one calculator to another.)

- Demonstrate the procedures for a) linking the TI-82 graphic calculator, CBL unit, and motion detector probe, and b) setting up the physical model for the pendulum experiment.

- Perform one trial of the experiment by using a TI-82 overhead unit.

- Establish groups of three or four students.

- Give students STUDENT INSTRUCTION SHEET and STUDENT ACTIVITY SHEET.

- Give students IN CONCLUSION... worksheet to complete after data and observations have been recorded.

Evaluation:

The teacher will circulate around the classroom to make sure all students are on task and following appropriate procedures. The activity and conclusion sheets will be collected and evaluated. Teachers may also have each group use overhead transparencies to present their data and graphs to the class.

Extension/Follow Up:

- Have students investigate whether or not changes in the weight of a pendulum will cause sinusoidal transformations.
- Have students investigate other real-world phenomena that are sinusoidal in nature.

Name of Developers, Locations, Phone Numbers:

Nancy Ferguson
Fallston High School
Harford County, MD

Tammy L. Mundie
Hereford High School
Baltimore County, MD

TEACHER NOTES
"JUST A MATTER OF TIME"

1. It is highly advisable that you perform the experiment on your own first before presenting a demonstration to the class.

2. It is suggested that a copy of the TICTOC program be linked to each student's calculator the day before the experiment.

3. The motion detector probe must be linked with the CBL through the sonic channel.

4. To avoid interference while the motion detector is collecting data, it is best to use two chairs (each on a desk) to support a meter stick from which to hang the pendulum. Use the third desk and/or chair (books may also be used) to support the motion detector.

5. After hanging the pendulum (through the tab on the can), be sure that the motion detector is level and approximately the same height from the floor as the soda can. The height of the motion detector needs to be adjusted every time the length of the pendulum is changed.

6. The student responsible for keeping time must do so for ten complete cycles. This will be independent of the time the motion detector is collecting data. In other words, the motion detector will generally shut off before ten full swings of the pendulum have been recorded.

7. Suggest to the students that when they change the length of the pendulum they should do so by at least 15 cm to get significant changes in their data.

8. While demonstrating the experiment to the students, emphasize that the motion detector will pick-up motion within a 15° angle from the horizontal. Therefore, the student starting the pendulum in motion needs to step out of the way of the motion detector before it is activated.

9. Yucky data constitutes any graph that does not appear sinusoidal.

Name:______________________ **Date:**_________________

Group #:___________________

STUDENT INSTRUCTION SHEET
"JUST A MATTER OF TIME"

Overview: By varying the length of a pendulum you will be able to investigate transformations of sinusoidal curves.

Materials Required:
- TI-82 graphics calculator with cable link
- CBL unit
- Vernier motion detector probe
- Pendulum unit consisting of string and an empty soda can
- Digital watch OR stopwatch
- Tape
- Meter stick
- Three chairs & desks
- Program TICTOC for TI-82
- STUDENT ACTIVITY SHEET
- IN CONCLUSION... worksheet

Procedures:

1. Before the data collection begins, use your tape and meter stick to mark two positions on the floor. These marks will represent the distance between your motion detector and the original position of your pendulum and should be between 75 and 120 centimeters. Keep this distance constant throughout this experiment, and record it on your activity sheet.

2. Hang a pendulum as instructed by your teacher. Make sure the bottom of the pendulum is at least 30 centimeters above the floor at all times. Keep this length constant for three trials, and record it on your activity sheet.

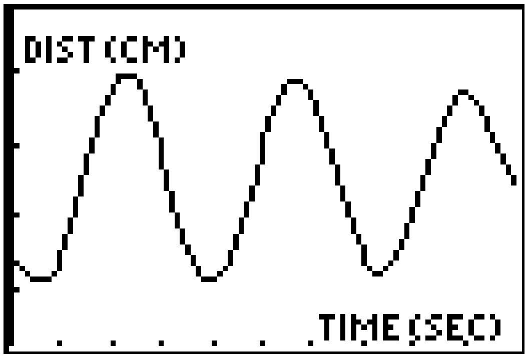
3. Assign to each individual in your group one of the following tasks:
   a. run the TICTOC program
   b. swing the pendulum
   c. time ten complete cycles of the pendulum

4. One cycle of the pendulum consists of the motion for one complete swing back and forth. During this activity, you will use a stopwatch to time the pendulum for ten complete cycles. The easiest way to do this is to begin the stopwatch when the pendulum "bob" is farthest from the detector and count one cycle when it returns to that spot. Record the time for ten cycles on your activity sheet.
   Note: Allow the pendulum to swing for a few seconds before timing the cycles. This will give the pendulum time to get into its natural rhythm.

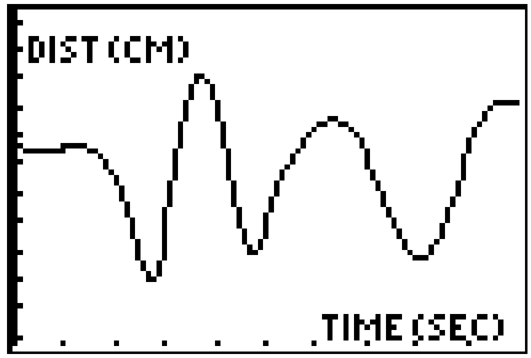
5. Run the TICTOC program on your TI-82 calculator and follow the instructions that are given. You do not need to measure the displacement of the pendulum "bob" from its original position.
   Note: The timer should be timing ten cycles as the TICTOC program runs.

6. If at any time you are dissatisfied with your results, press $\boxed{\text{CLEAR}}$ $\boxed{\text{ENTER}}$ to collect new data.

<u>Example of Good Data</u>



<u>Example of Yucky Data</u>



7. From your graph, calculate the amplitude and phase shift, and record this information on your activity sheet.

8. Use the time for ten complete cycles to *determine* the period and record this on your activity sheet.

9. Recall that the general form of a sinusoidal curve is **Y = A cos B(X – C) + D**. Enter this equation into $\underline{Y_1}$. Then use your calculator to store the results obtained from above into **A**, **B**, **C**, and $\boxed{\textbf{D}}$. Then graph this function.
Note: This graph should closely correspond to that produced by your data.

10. Repeat steps 4 through 9 two more times to complete the table.

11. To complete **Tables 2 & 3** on your activity sheet, you will need to change the length of your pendulum, then repeat steps 2 through 10.

12. After completing the activity sheet answer all questions on **IN CONCLUSION...**

Name: ______________________ **Date:** _________________

Group #: ___________________

STUDENT ACTIVITY SHEET
"JUST A MATTER OF TIME"

Distance from motion detector to the original position of the pendulum: _________ cm

TABLE 1:

Length of pendulum: _________ cm

	Time of 10 Cycles	Period	2π/period	Amplitude	Phase Shift	Vertical Shift
Trial 1						
Trial 2						
Trial 3						

TABLE 2:

Length of pendulum: _________ cm

	Time of 10 Cycles	Period	2π/period	Amplitude	Phase Shift	Vertical Shift
Trial 1						
Trial 2						
Trial 3						

TABLE 3:

Length of pendulum: _______ cm

	Time of 10 Cycles	Period	2π/period	Amplitude	Phase Shift	Vertical Shift
Trial 1						
Trial 2						
Trial 3						

Name:______________________ **Date:**__________________

Group #:____________________

IN CONCLUSION...
"JUST A MATTER OF TIME"

1. Choose one trial from each table, and write the corresponding equations below.

   Y_1 = ______________________________

   Y_2 = ______________________________

   Y_3 = ______________________________

2. Graph all three functions in the grid below (quadrant I only). Label each axis and function (possibly using different colors to distinguish between each function). Indicate your viewing rectangle.

[_______, _______] by [_______, _______]

3. Based on the data you collected in this experiment and your graph above, in what way does varying the length of the pendulum effect the sinusoidal curve? Be specific.

4. Using your equations from question 1, explain how the values of **A**, **B**, **C**, and **D** would change if you use a sine curve to fit the data?

5. Write the equations which model these motions in the form $y = A \sin B(X - C) + D$.

   Y_4 = ______________________________

   Y_5 = ______________________________

   Y_6 = ______________________________

   Test your prediction by graphing equations Y_1 through Y_6 on your calculator.

6. Explain the *physical* interpretations of the variables **A**, **B**, **C**, and **D** as they relate to the swinging pendulum and the equation $Y = A \cos B(X - C) + D$.

variable	graphical transformation	physical interpretation
A		
B		
C		
D		

7. What were some difficulties you encountered in your efforts to collect good data? Be specific.

8. Thelma Timex inherited her great-grandfather's grandfather clock. Obviously it hasn't "kept on ticking" because it's 45 minutes slow. Twenty-four hours after adjusting the hands to display the correct time, she notices that the clock is again 45 minutes slow. Remembering her fun in working with trigonometry, she realizes that she needs to adjust the length of the pendulum. What adjustment must Thelma make? Explain your answer.

Name: SAMPLE **Date:** ____________

Group #: ____________

STUDENT ACTIVITY SHEET - SAMPLE DATA
"JUST A MATTER OF TIME"

Distance from motion detector to the original position of the pendulum: 100 cm

TABLE 1:

Length of pendulum: 53 cm

	Time of 10 Cycles	Period	2π/period	Amplitude	Phase Shift	Vertical Shift
Trial 1	14.27	1.427	4.403	4.5	0.9	100
Trial 2	14.39	1.439	4.366	5.2	0.8	100
Trial 3	14.29	1.429	4.397	5.4	0.8	100

TABLE 2:

Length of pendulum: 37 cm

	Time of 10 Cycles	Period	2π/period	Amplitude	Phase Shift	Vertical Shift
Trial 1	11.99	1.199	5.240	5.7	0.6	100
Trial 2	12.26	1.226	5.125	7.6	0.7	100
Trial 3	12.24	1.224	5.133	5.5	0.6	100

TABLE 3:

Length of pendulum: 70 cm

	Time of 10 Cycles	Period	2π/period	Amplitude	Phase Shift	Vertical Shift
Trial 1	17.49	1.749	3.592	8.7	1.2	100
Trial 2	17.92	1.792	3.506	10.6	1.6	100
Trial 3	17.57	1.757	3.576	10.8	1.5	100

Name: SAMPLE KEY **Date:** ____________

Group #: ____________

IN CONCLUSION... – ANSWER KEY FOR SAMPLE DATA
"JUST A MATTER OF TIME"

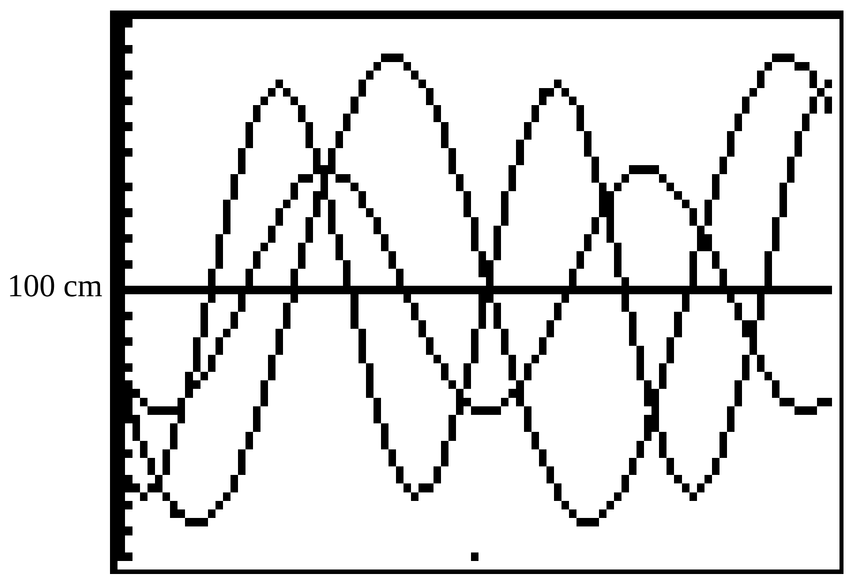
1. Choose one trial from each table, and write the corresponding equations below.

   $\mathbf{Y_1}$ = $4.5 \textit{ cos } 4.403(x - 0.9) + 100$

   $\mathbf{Y_2}$ = $7.6 \textit{ cos } 5.125(x - 0.7) + 100$

   $\mathbf{Y_3}$ = $8.7 \textit{ cos } 3.592(x - 1.2) + 100$

2. Graph all three functions in the grid below (quadrant I only). Label each axis and function (possibly using different colors to distinguish between each function). Indicate your viewing rectangle.



[0 , 3.14] by [90 , 110]

3. Based on the data you collected in this experiment and your graph above, in what way does varying the length of the pendulum effect the sinusoidal curve? Be specific.

   ANSWER: *The length of the pendulum changes the period of the sinusoidal curve. In other words, the time to complete one complete cycle (or one full swing) changes. A longer pendulum takes more time to complete one cycle, thus having a greater period.*

4. Using your equations from question 1, explain how the values of **A**, **B**, **C**, and **D** would change if you use a sine curve to fit the data?

   ANSWER: *The values of A, B, and D would not change. The only change between the sine and cosine curve would involve the phase shift. Therefore, to determine the phase shift for the sine curve simply divide the period of the cosine curve by 4 and subtract from the phase shift of the cosine curve.*
   Note*: The cosine and sine curves with a period of* 2π *are related by a shift of* $\pi/2$*, (which is 1/4 of the period).*

5. Write the equations which model these motions in the form $\mathbf{y = A \sin B(X - C) + D}$.

   $\mathbf{Y_4}$ = $4.5 \sin 4.403(x - 0.5) + 100$

   $\mathbf{Y_5}$ = $7.6 \sin 5.125(x - 0.4) + 100$

   $\mathbf{Y_6}$ = $8.7 \sin 3.592(x - 0.8) + 100$

   Test your prediction by graphing equations $\mathbf{Y_1}$ through $\mathbf{Y_6}$ on your calculator.

6. Explain the *physical* interpretations of the variables **A**, **B**, **C**, and **D** as they relate to the swinging pendulum and the equation $\mathbf{Y = A \cos B(X - C) + D}$.

variable	**graphical transformation**	**physical interpretation**
A	*amplitude*	*distance pendulum is displaced from its original position*
B	2π*/period*	*related to the period and is determined by dividing* 2π *by the time to complete one full swing*
C	*phase shift*	*the position of the pendulum in its swing when the motion detector is first activated*
D	*vertical shift*	*the horizontal distance from the motion detector to the pendulum in its original position*

7. What were some difficulties you encountered in your efforts to collect good data? Be specific.

   Answers may vary.

8. Thelma Timex inherited her great-grandfather's grandfather clock. Obviously it hasn't "kept on ticking" because it's 45 minutes slow. Twenty-four hours after adjusting the hands to display the correct time, she notices that the clock is again 45 minutes slow. Remembering her fun in working with trigonometry, she realizes that she needs to adjust the length of the pendulum. What adjustment must Thelma make? Explain your answer.

   ANSWER: *Thelma must* ***shorten*** *the length of the pendulum. This would increase the number of swings the pendulum will complete in a 24 hour period.*

[1] Brueningsen, C., Bower, B., Antinone, L., & Brueningsen, E., Real World Math with the CBL System --Programs for the TI-82, Texas Instrument Inc., 1994.